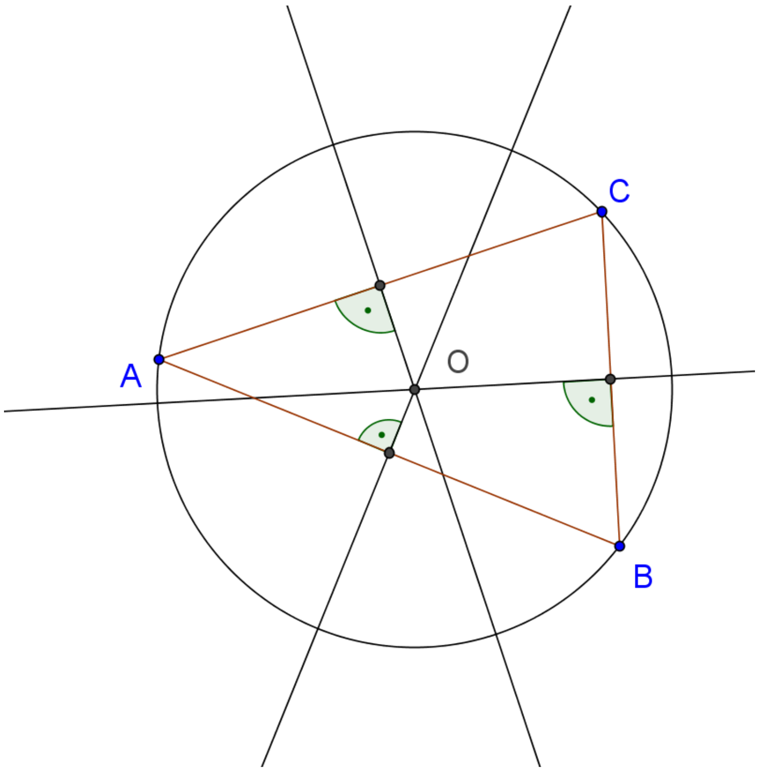
Triangles

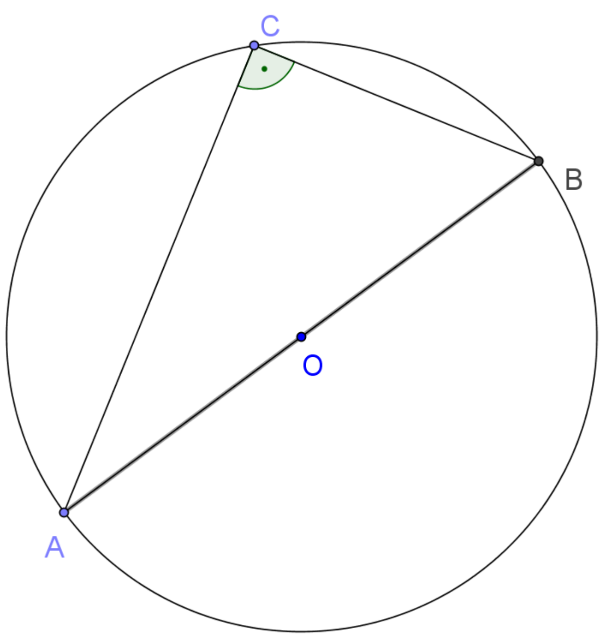
The perpendicular bisectors of the sides



- The three perpendicular bisectors of the sides of a triangle meet at a point (O)
- The point O is the center of the circumscribed circle of the triangle.

The theorem of Thales

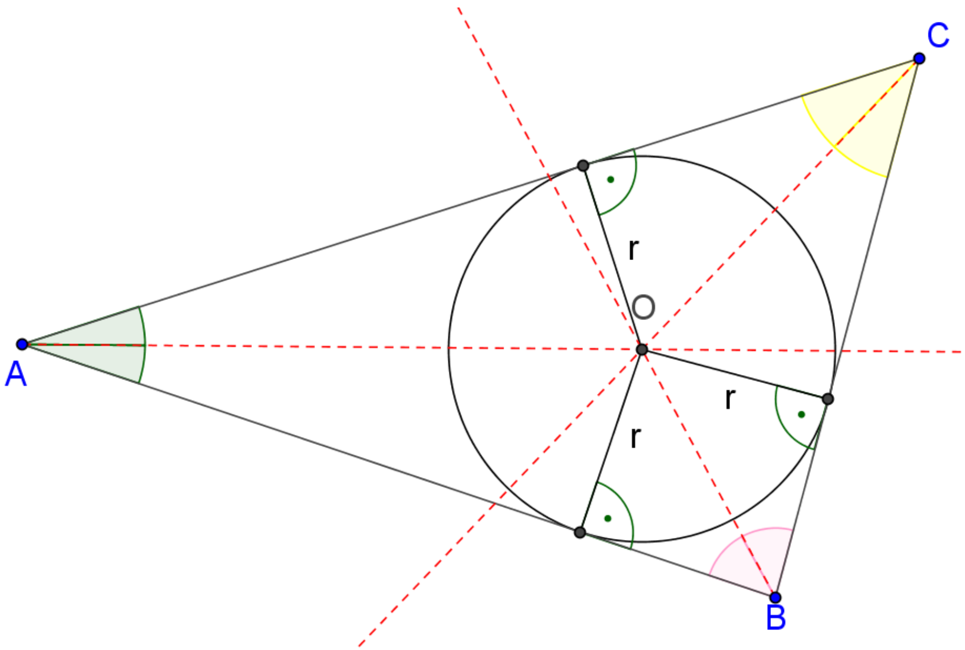
If we connect the two end-points (A, B) of the diameter of a circle with any other point of the circle, then we get a right triangle.



$\Downarrow$

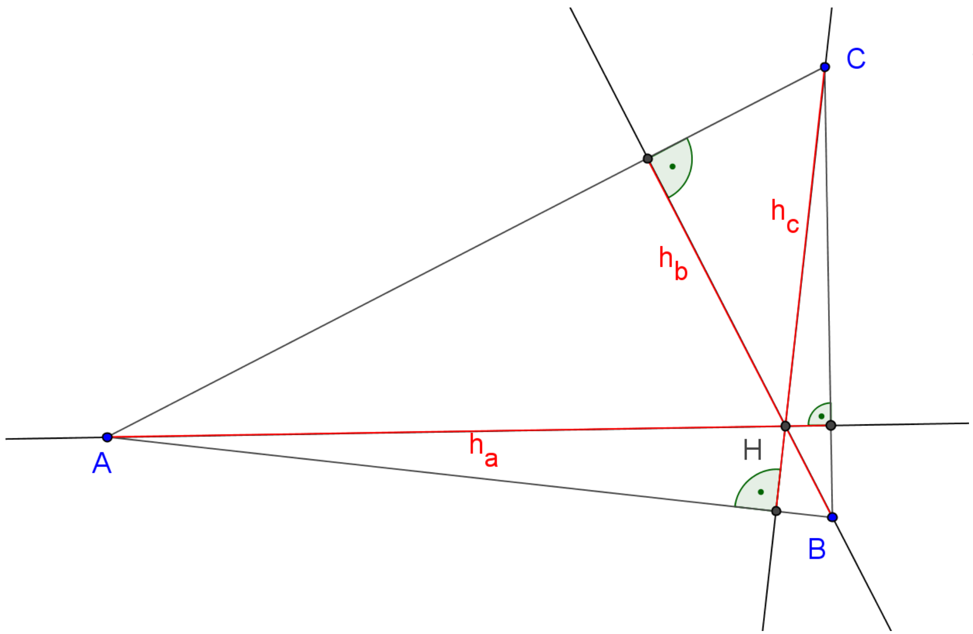
The midpoint of the hypotenuse is center of the circumscribed circle of the right triangle.

The angle bisectors of a triangle



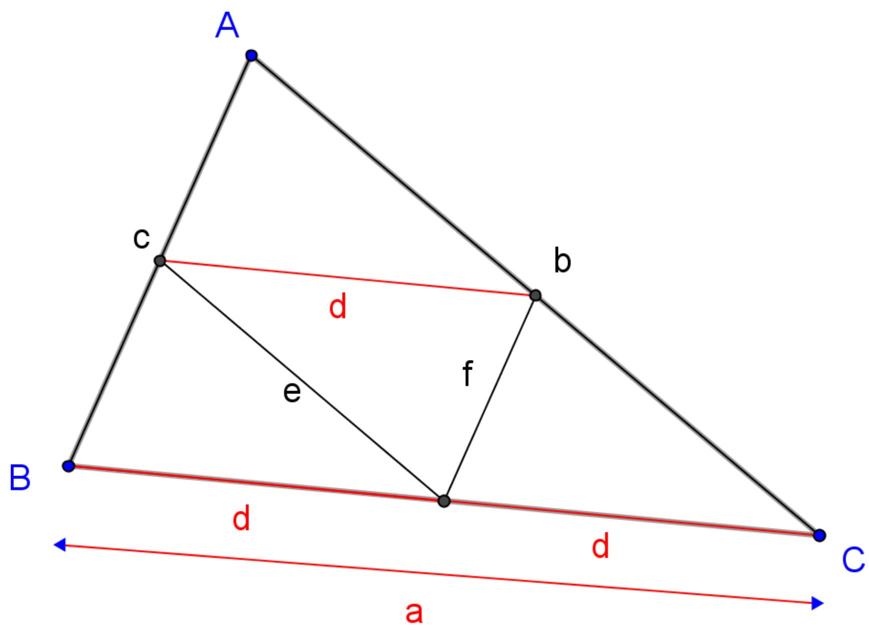
- The angle bisectors of a triangle intersect each other at one point.
- This point (O) is the center of the inscribed circle of the triangle.

The altitudes (heights) of a triangle



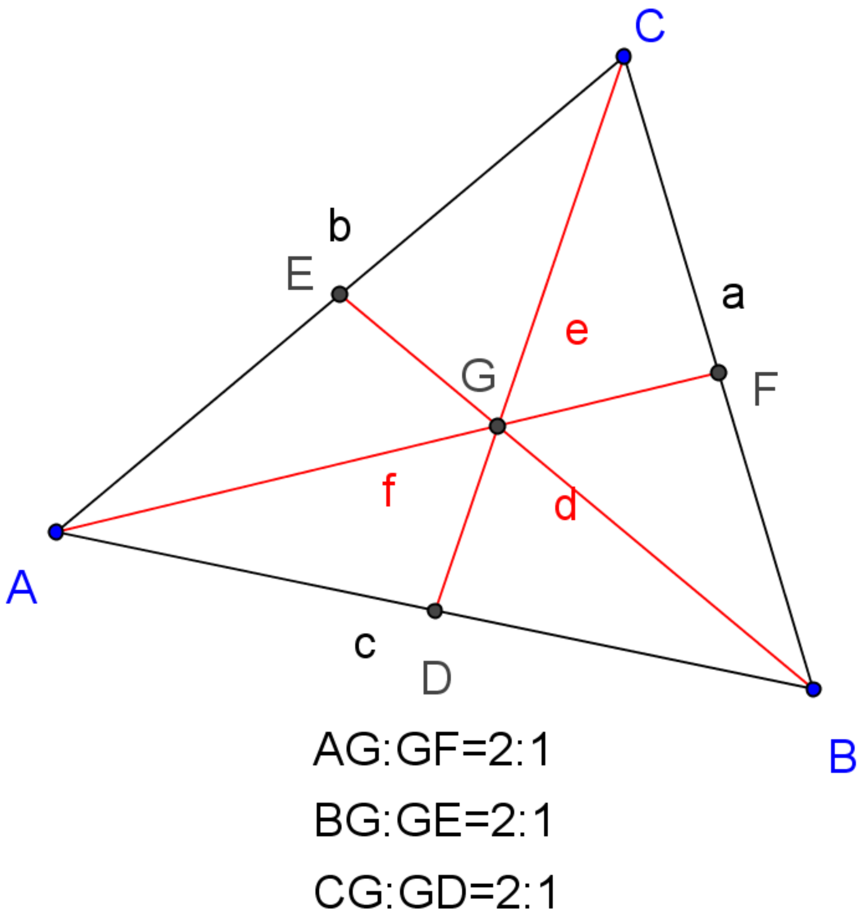
- An altitude (or height) of a triangle is a segment which passes through a vertex of the triangle and is perpendicular to the side opposite this vertex.
- Orthocenter of the triangle: H

Midlines of a triangle



- The segment connecting the midpoint of two sides of a triangle is a midline.
- Each midline is parallel and half as long as to the third side of the triangle.

Medians of a triangle



- A median of a triangle is a segment from a vertex of the triangle to the midpoint of the opposite side.
- The medians of a triangle are concurrent. The common point is the centroid, which divides the medians in the ratio 2:1.

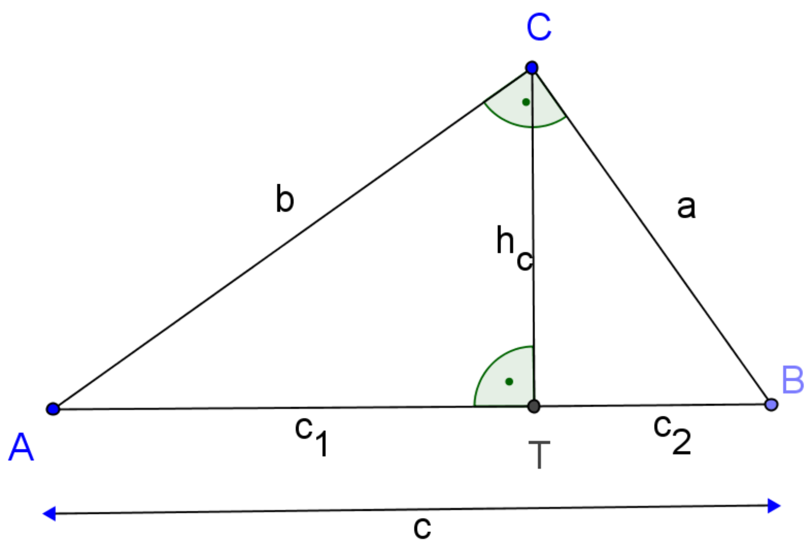
Congruent triangles

- They will have exactly the same three sides and exactly the same three angles.
- The equal sides and angles may not be in the same position.
- There are four ways to find if two triangles are congruent:
  1. SSS *(side, side, side)*
  2. SAS *(side, angle, side)*
  3. ASA *(angle, side, angle)*
  4. SSA *(side, side, angle opposite the longer side)*

Similar triangles

- Triangles are similar if they have the same shape, but not necessarily the same size.
- In similar triangles, the sides facing the equal angles are always in the same ratio.
- Triangles are similar if:
  1. AAA *(angle, angle, angle)*
     All three pairs of corresponding angles are the same.
  2. SSS in same proportion *(side, side, side)*
     All three pairs of corresponding sides are in the same proportion
  3. SAS *(side, angle, side)*
     Two pairs of sides in the same proportion and the included angle equal.
  4. SSA *(side, side, angle)*
     Two pairs of sides in the same proportion and the angles opposite the longer side are the same.

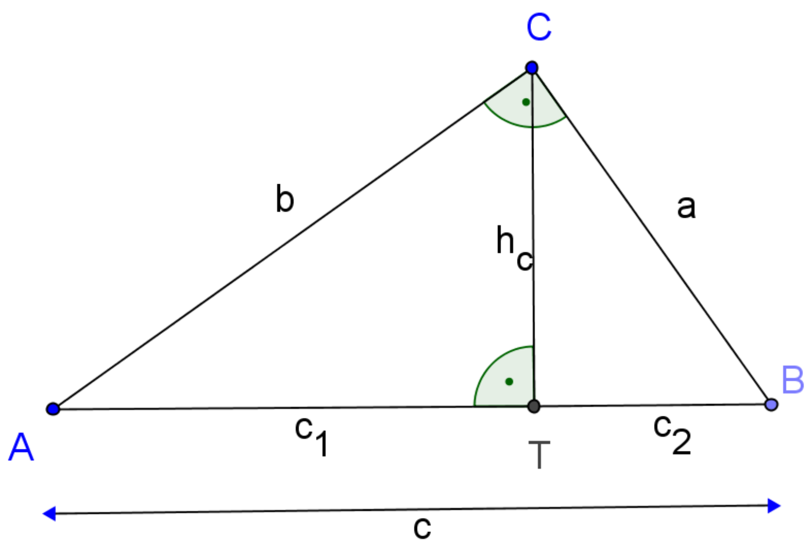
Right Triangle Altitude Theorem



$$h_c = \sqrt{c_1 \cdot c_2}$$

- The altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

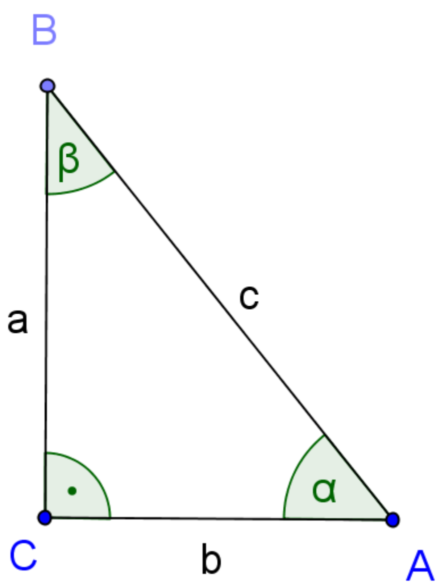
Right Triangle Leg Theorem



$$b = \sqrt{c_1 \cdot c}$$
$$a = \sqrt{c_2 \cdot c}$$

- In right triangles, leg is the geometrical mean of the hypotenuse and the orthogonal projection of the leg on the hypotenuse.

Similar right triangles



- Two right triangles are similar if one pair of acute angles are equal.

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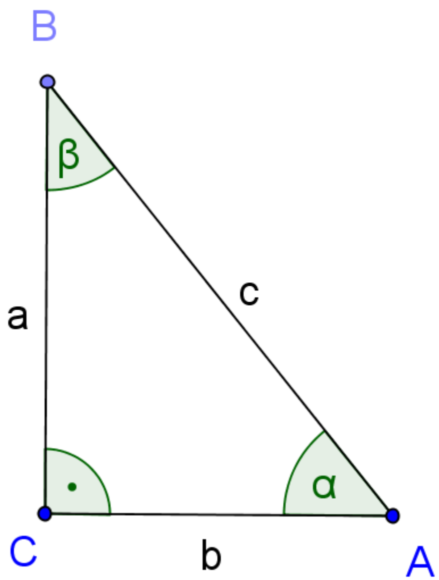
- The acute angle of a right triangle is determined by the ratio of two sides.

The trigonomic functions of an acute angle: sine, cosine, tangent



- $\sin\alpha = \frac{a}{c} \left(= \frac{opposite\ leg}{hypotenuse} \right)$
- $cos\alpha = \frac{b}{c} \left(= \frac{adjacent\ leg}{hypotenuse} \right)$
- $tan\alpha = \frac{a}{b} \left(= \frac{opposite\ leg}{adjacent\ leg} \right)$

Connections between the trigonomic functions of an acute angle

- $sin\alpha = cos(90° - \alpha)$
- $cos\alpha = sin(90° - \alpha)$
- $tan\alpha = \frac{sin\alpha}{cos\alpha}$
- $sin^2\alpha + cos^2\alpha = 1$

The exact values of trigonometric functions of some acute angles

	sin	cos	tan
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$