AUTOMATIC NEURAL NETWORK SYSTEM FOR VORTICITY OF SQUARE CYLINDERS WITH DIFFERENT CORNER RADII

MOSTAFA Y. EL-BAKRY, A. A. EL-HARBY, AND G. M. BEHERY

Abstract. The neural networks (NNs) simulation has been designed to simulate and predict the vortex wavelength $\lambda_x$, lateral vortex spacing $\lambda_y$, and normalized maximum vorticity at the vortex center near the wake of square cylinders with different corner radii. The system was trained on the available data of the three cases, although this data is very little. Therefore, we designed the system to work in automatic way for finding the best network that has the ability to have the best test and prediction. The proposed system shows an excellent agreement with that of an experimental data in these cases. The technique has been also designed to simulate the other distributions not presented in the training set and predicted them with effective matching.

Keywords: Neural Networks, RPROP, Backpropagation, vorticity, square cylinder.

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1. Introduction

Flow passing a cylindrical body with a corner modification has attracted a great deal of attention in the literature because of its practical significance in engineering, e.g., in the designed of tall buildings, tower structure, suspension bridges, etc. [1-2] investigated numerically and experimentally the aerodynamic forces on square cylinders. Zheng and Dalton [3] studied numerically the corner effect in an oscillatory flow. Recently, Dalton and Zheng [4] presented numerical results for a uniform approach flow past square and diamond cylinders, with and without corner modifications at Reynolds number 250 and 1000. Similar studies emphasizing corner effects were also conducted by [5-8]. These investigations largely focused on the effect of corner radii on the aerodynamic or hydrodynamic...
characteristics, such as drag and lift forces, of bluff bodies. How the corner variation may alter the near wake, however, has yet to be sufficiently documented, particularly in the base region.

Recently, Hu, Zhou and Dalton [9] studied both qualitative and quantitative vortex flow fields near wake and then introduce the corner effects on the near wake flow structure, complementary the data in the literature.

The present work introduce the artificial neural network for modelling the vortex flow fields; vortex wavelength $\lambda^*_x$, lateral vortex spacing $\lambda^*_y$ and normalized maximum vorticity at the vortex center, near wake of square cylinders which represented by $x/d$, where $x$ is the distance from center of cylinder and $d$ is the characteristic dimension of the cylinder. Four cylinders have the same characteristic dimension $d=12.7\text{mm}$ but have different corner radius $r$, therefore, we have square cylinder of $r/d=0$ and three square cylinder with rounded corners $r/d=0.157, 0.236$ and $0.5$ (circular cylinder).

Neural networks are widely used for solving many problems in most science problems of linear and non-linear cases [10-18]. Neural network algorithms are always iterative, designed to step by step minimise (targeted minimal error) the difference between the actual output vector of the network and the desired output vector, examples include the Backpropagation (BP) algorithm [19-21], and the Resilient Propagation (RPROP) algorithm [22-24].

BP is the most widely used algorithm for supervised learning with multi-layered feed-forward networks [25], and it is very well known, while the RPROP algorithm is not well known and described in some detail in section 2.

The data obtained by [9] is chosen to be carried out using the neural networks depending on the BP and RPROP algorithms. The RPROP algorithm was faster than the BP [26-27]. Therefore, the RPROP is chosen to be carried out in this study. The present work offers an efficient neural network system that is used to predict the unknown data of the normalized maximum vorticity ($\omega^*_z$), the vortex wave length ($\lambda^*_x$), and the lateral vortex spacing ($\lambda^*_y$) near wake of square cylinders at different corner radii. The rest of paper is organized as follows; Section 2 describes the trained NN. Section 3 presents the proposed system. Section 4 shows the obtained results. Finally, Section 5 concludes the work.

2. Trained neural networks

Neural networks consist of a number of units (neurons) which are connected by weighted links. These units are typically organised in several layers, namely an input layer, one or more hidden layers, and an output layer. The input layer receives an external activation vector, and passes it via weighted connections to the units in the first hidden layer. Figure (1) shows input layer with $R$ elements, one hidden layer with $S$ neurons, and output layer with one element. Each neuron in the network is a simple processing unit that computes its activation $y_i^{(1)}$ with respect to its incoming excitation, the so-called net input $net_i$:
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\[
\text{net}_i = \sum_{j \in \text{pred}(i)} s_j \omega_{ij} - \theta_i
\]  

(1)

Where denotes the set of predecessors of unit i, \( \omega_{ij} \) denotes the connection weight from unit to unit, and \( \theta_i \) is the unit bias value. The activation of unit i, \( y_i^{(1)} \) is computed by passing the net input through a non-linear activation function. The log-sigmoid function is applied in the proposed work as follows.

\[
y_i^{(1)} = f_{\text{log sig}}(\text{net}_i) = \frac{1}{1 + e^{-\text{net}_i}}
\]

(2)

In the RPROP algorithm, each weight \( \omega_{ij} \) is computed by its individual update-value \( \Delta_{ij}^{(t)} \), which determines the size of the weight-update. This adaptive update-value evolves during the learning process based on its local sight on the error function \( E \), according to the following learning-rule [22].

FIGURE 1. Network architecture for one hidden layer
\[ \Delta^{(t)}_{ij} = \begin{cases} 
\eta^+ \cdot \Delta^{(t-1)}_{ij}, & \text{if } \frac{\partial E}{\partial w_{ij}} > 0 \\
\eta^- \cdot \Delta^{(t-1)}_{ij}, & \text{if } \frac{\partial E}{\partial w_{ij}} < 0 \\
\Delta^{(t-1)}_{ij}, & \text{else} 
\end{cases} \tag{3} \]

where \(0 < \eta^- < 1 < \eta^+\). The size of the weight change is exclusively determined by the weight-specific update-value \(\Delta^{(t)}_{ij}\). Every time the partial derivative of the corresponding weight changes its sign, the update-value is decreased by the factor \(\eta\). This indicates that the last update was too big and the algorithm jumped over a local minimum. On the other hand, if the derivative retains its sign the update-value is slightly increased by the factor \(\eta\) in order to accelerate convergence in shallow regions. Once the update-value for each weight is adapted, the weight-update is changed as follows: if the derivative is positive (increasing error) the weight is decreased by its update-value, if the derivative is negative, the update-value is added. Then, the weights are updated as in equation (5) using update-values from equation (4).

\[ \Delta^{(t)}_{ij} = \begin{cases} 
-\Delta^{(t)}_{ij}, & \text{if } \frac{\partial E}{\partial w_{ij}} > 0 \\
\Delta^{(t)}_{ij}, & \text{if } \frac{\partial E}{\partial w_{ij}} < 0 \\
0, & \text{else} 
\end{cases} \tag{4} \]

\[ \omega^{(t+1)}_{ij} = \omega^{(t)}_{ij} + \Delta^{(t)}_{ij} \tag{5} \]

As mentioned in the Section 1, the RPROP algorithm was faster than the BP, the main reason for the success using this algorithm is that the size of weight-step is only dependent on the sequence of signs, not on the magnitude of the derivative as showed by Riedmiller and Braun [26]. The RPROP algorithm has fewer parameters that need to be evaluated and promises to provide the same performance as an optimally trained network using the BP algorithm.

3. Proposed system

The studied problem consists of three depended-parts, the first one is the normalized maximum vorticity \(\omega^*_x\) with the wake of cylinder, the second is the vortex wave length \(\lambda^*_x\) dependence on the wake of cylinder the wake of cylinder and the third is the lateral vortex spacing \(\lambda^*_y\) dependence on the wake of cylinder. Each part contains four groups of data corresponding to four different corner radii of cylinders. These are square cylinder of \(r/d=0\) and three square cylinder with rounded corners \(r/d=0.157, 0.236\) and 0.5 (circular cylinder). Each group has some samples as specified in [9]. The group data which for \(r/d=0.236\) is specified to be predicted for each part, while the other three groups are chosen
as patterns for training. The three groups for each part are prepared as input patterns of the proposed neural network system.

Our problem has two inputs and single output in each part, because there is only one target value associated with each input vector; see figure (2). These parts are:

1. part (1) has two inputs corner radius \( r/d \) and distance near wake \( x/d \) and single output which is the normalized maximum vorticity \( \omega_z^* \)
2. part (2) has two inputs corner radius \( r/d \) and distance near wake \( x/d \) and single output which is the vortex wave length \( (\lambda_x^*) \)
3. part (3) has two inputs corner radius \( r/d \) and distance near wake \( x/d \) and single output which is the lateral vortex spacing \( (\lambda_y^*) \)

The available data of the three cases is very little and not enough for good training. The number of elements in each group is different. This affected the training and prediction process, due to increasing the number of experiments. Therefore, we have designed the system to work in automatic way for 1000 experiments, this process stops when the best network is obtained. If the last NN-experiment is reached, the number of neurons of the hidden layer is increased and the process of new 1000 NN-experiment starts again. The system is continuous until excellent training and prediction is reached. The details of the proposed system are shown in figure (3).

We have preferred to use the same neural network architecture in the three cases: the normalized maximum vorticity, the vortex wave length, and the lateral vortex spacing; see figure (4). The chosen algorithm was first trained up to 5000 epochs for the three cases. The obtained best networks are reached at 1000 epochs for the normalized maximum vorticity part, 564 epochs for the vortex wave length part, and 554 epochs for the lateral vortex spacing part as described in figure(5). After the training, it was noticed that the RPROP algorithm using one hidden layer was very effective using log-sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. It was found that, one hidden layer and 10 neurons are enough for reaching the optimal solution as specified in figure (4). We first set up the network with random weights and biases values.

Where \( IW(1,1) \) represents the input weights, \( IW(2,1) \) means the layer weights, \( b(1) \) is the biases of the input layer, and \( b(2) \) be the biases of the output layer. The obtained weights and biases of the best trained network for the normalized maximum vorticity \( (\omega_z^*) \), the vortex wave length \( (\lambda_x^*) \), and the lateral vortex spacing \( (\lambda_y^*) \) are shown in Table 1 (Appendix A).

4. Results
The proposed system was applied and simulated to the data of the normalized maximum vorticity, the vortex wave length, and the lateral vortex spacing near wake of square cylinders, with one hidden layer.

The chosen neural network was trained on three cases of different corner radii ratios r/d. The values of r/d are 0, 0.157, 0.236, and 0.5. The performances of the obtained networks are shown in figure (5). The obtained networks were tested for choosing the best one. This network was tested on the above mentioned three cases and used for predicting the case at r/d value, 0.236. Figure (6) shows the neural networks results of the three cases training and one predicted case the normalized maximum vorticity which denoted by $\omega^*_z$ with wake distance ratio x/d. Figure (7) shows also the neural networks results of the three cases training and one predicted case for the vortex wave length which denoted by $\lambda^*_x$ with x/d. Figure (8) shows also the neural networks results of the three cases training and one predicted case for the lateral vortex spacing which denoted by $\lambda^*_y$ with x/d. It was observed that these figures illustrate an excellent performance in the four cases (the training and prediction). These results of the dependence of the normalized maximum vorticity, the vortex wave length, and the lateral vortex spacing on x/d at different values of corner radii are presented in the following three subsections.

4.1 Normalized maximum vorticity

The following figure shows the three cases tested and one predicted data of the normalized maximum vorticity compared to the experimental data.

4.2 Vortex wave length
The following figure shows the three cases tested and one predicted data of the vortex wave length compared to the experimental data.

4.3 Lateral vortex spacing
The following figure shows the three cases tested and one predicted data of the lateral vortex spacing compared to the experimental data.

5. Conclusion

We have designed the system to work in automatic way for finding the best network that has the ability to have the best test and prediction. This system did many tries to find the best network used low number of hidden layers and neurons. It was found that, one hidden layer with 10 neurons are enough for reaching the optimal solution.

The proposed system shows an excellent agreement with that of an experimental data in the three cases of normalized maximum vorticity, vortex wave
length, and lateral vortex spacing problems. The NN technique has been also designed to simulate the other distributions not presented in the training set and matched them effectively.

The (NNs) simulation using RPROP algorithm is a powerful mechanism for prediction the normalized maximum vorticity, vortex wavelength, and lateral vortex spacing near wake of square cylinder of any suggested corner radii.
The system has the ability to store the obtained networks including the weighted and biases values; see table (1). Thus provides the system to make the test and prediction process without retraining again.

**Appendix**
### Normalized maximum vorticity

<table>
<thead>
<tr>
<th>Weights</th>
<th>Biases</th>
</tr>
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<tbody>
<tr>
<td>( IW(1,1) )</td>
<td>( LW(2,1)^f )</td>
</tr>
<tr>
<td>(-17.4615)</td>
<td>(-2.901)</td>
</tr>
<tr>
<td>(-22.6727)</td>
<td>(-2.6198)</td>
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<tr>
<td>(14.8643)</td>
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<tr>
<td>(-7.7975)</td>
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<tr>
<td>(-30.3999)</td>
<td>(0.76515)</td>
</tr>
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<td>(-6.1247)</td>
<td>(1.8982)</td>
</tr>
<tr>
<td>(-1.3998)</td>
<td>(-1.3589)</td>
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</table>

### Vortex wave length

<table>
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<th>Biases</th>
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</thead>
<tbody>
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<td>( IW(1,1) )</td>
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<tr>
<td>(-29.435)</td>
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<td>(-17.7383)</td>
<td>(1.9386)</td>
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</table>

### Lateral vortex spacing

<table>
<thead>
<tr>
<th>Weights</th>
<th>Biases</th>
</tr>
</thead>
<tbody>
<tr>
<td>( IW(1,1) )</td>
<td>( LW(2,1)^f )</td>
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<tr>
<td>(27.1012)</td>
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<td>(6.3794)</td>
<td>(1.236)</td>
</tr>
<tr>
<td>(3.6715)</td>
<td>(-2.3096)</td>
</tr>
</tbody>
</table>
Where $LW^{(2,1)^T}$ represents the transpose of $LW^{(2,1)}$. All names in the above table were described in architecture of the proposed network; see figure (4).

References


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